Application of MLS and Haramard Trasnform in Audio Systems

Northwestern Polytechnic University
Graduate Seminar

Jahan Ghofraniha, Ph.D.

Outline

- Input/Output Modeling of Linear Systems.
- Significance of different Input Types.
- Maximum Length Sequence.
- Correlation Analysis and Hadamard Transform.
- Results.
- Conclusions.

Input/Output Modeling

- Input/Output characterization of linear systems is the most common approach to system modeling normally referred to as black box modeling.
- The quality of the output in revealing system behavior relies heavily on the richness of the input.

Input/Output Modeling

- In time domain, a linear system is characterized by its impulse response.
- A good candidate for an input to a system for identification purposes should have the following properties: -
- Rich in terms of frequency.
- Does not cause clipping (low crest factor).

Input Properties

- Does not upset the on-going operation of the system.
- Does not have a high degree of correlation and is not correlated with system noise.
- A good candidate for the input signal is white noise (high crest factor).
- Lot of systems are sensitive to noise (derivative components in the system).

Maximum Length Sequence

- One type of input that satisfies all these conditions is pseudo random binary sequence (PRBS) or Maximum Length Sequence (MLS).
- Will not disturb the system.
- MLS has a wide spectrum.
- Low degree of correlation.
- If designed properly will not cause clipping and nonlinear behavior (low crest factor).

MLS

- The generation of MLS is based on a concept in Mathematics known as primitive polynomials.
- These polynomials have the property that they can generate random bit recursively with very long periods. An example is shown below:

$$x^{18} + x^5 + x^2 + x^1 + 1$$

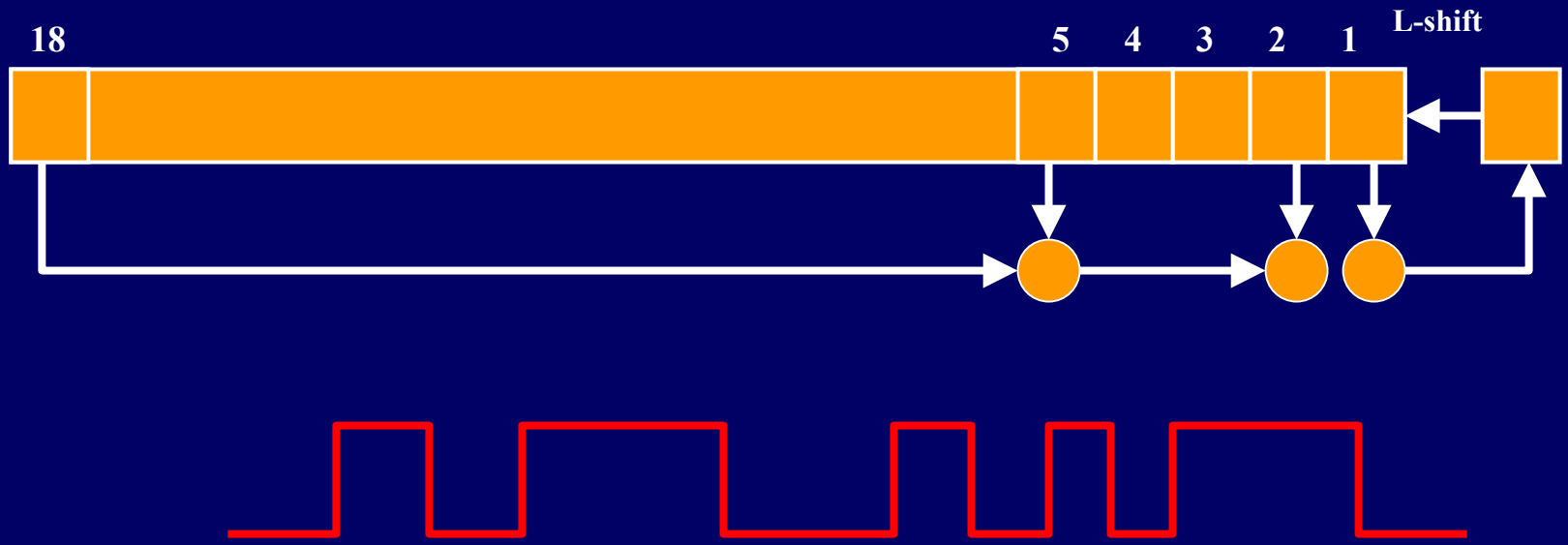
MLS Generation

- In order to start the sequence, we seed them with non-zero word and use the following formula to calculate the new bit. Every time a bit is generated we left shift all the words while assuming the shift register has length n. In the following n=18.

$$a_{new} = a_{18} \oplus a_5 \oplus a_2 \oplus a_1$$

MLS Generation

- A hardware block diagram representation of an MLS generator is shown below:



Output Analysis

- The purpose of the input/output analysis is to find the impulse response of the system.

$$h(n) = \frac{\phi_{input/output}}{\phi_{input}} = \frac{\frac{1}{n-1}\sum_{k=0}^{k=n-1} x(k).y(k+\tau)}{\sum_{k=0}^{k=n-1} x(k).x(k+\tau)}$$

Hadamard Matrix/Transform

- If in the cross-correlation formula, all the x's are either 1's and –1's, then the numerator reduces to summation and subtraction of shifted valued of the outputs.
- This process, in its simple form can be implemented with a Hadamard transform.
- A Hadamard matrix of order 2 is defined as:

Hadamard Matrix

$$H_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- In order to calculate Hadamard of order 2, we just need to replace each element of the new matrix with H_1 in the following manner:

$$H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix}$$

Hadamard Matrix Properties

- Symmetric.
- Orthogonal.
- Same number of 1's and –1's in all rows and columns except first.
- Self-inverting ($H_n^{-1} = (1/2^n)H_n$).
- There is a fast algorithm to implement Hadamard transform referred to as Fast Hadamard Transform or FHT, similar to FFT.

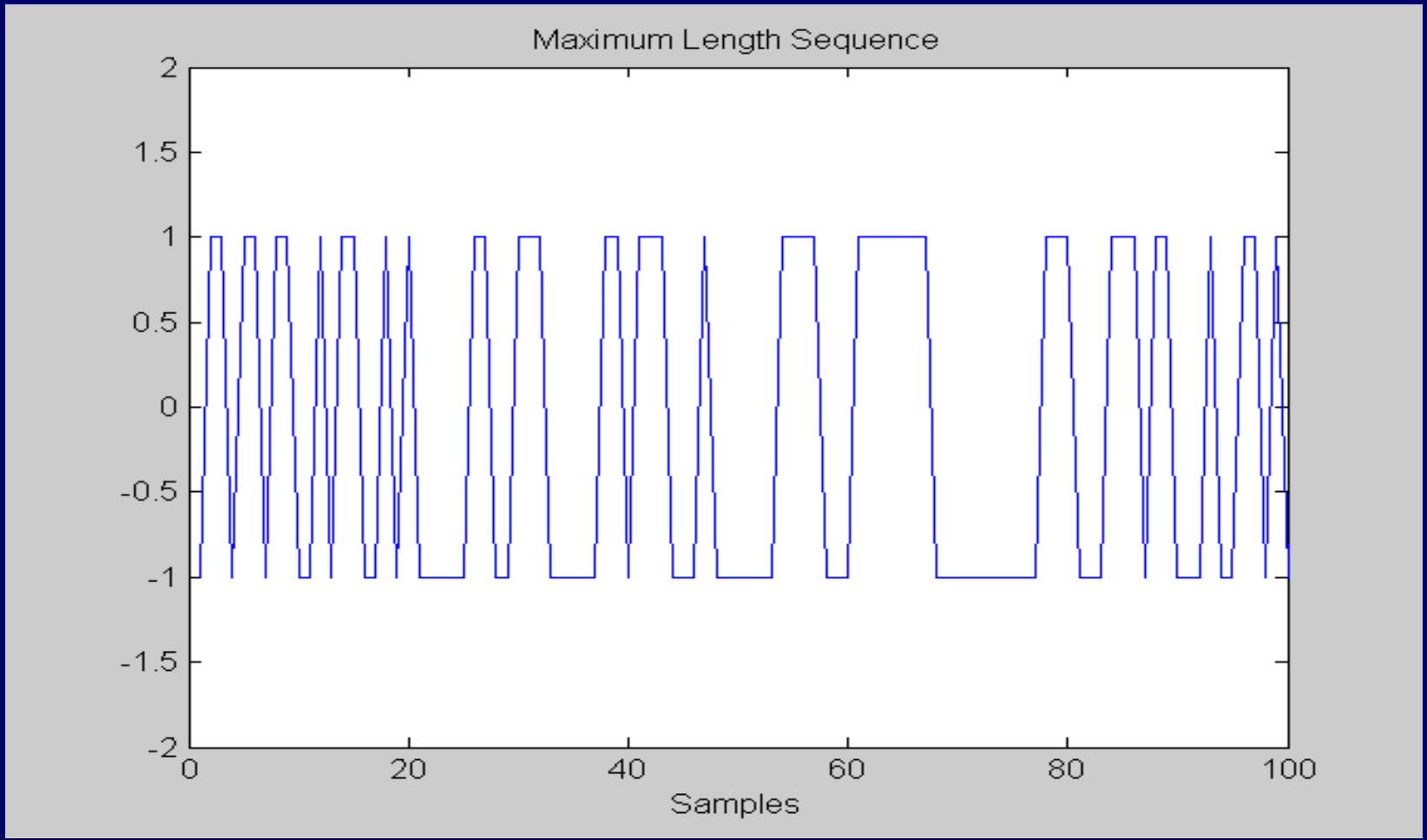
Hadamard Transform

- Hadamard transform has many applications in spectroscopy and communications such as data compression and channel coding and specifically in wireless communications (CDMA/IS-95).
- One draw back of FHT as compared to FFT is its sensitivity to phase shift.
- This might be one reason that FHT is not popular in spectral analysis.

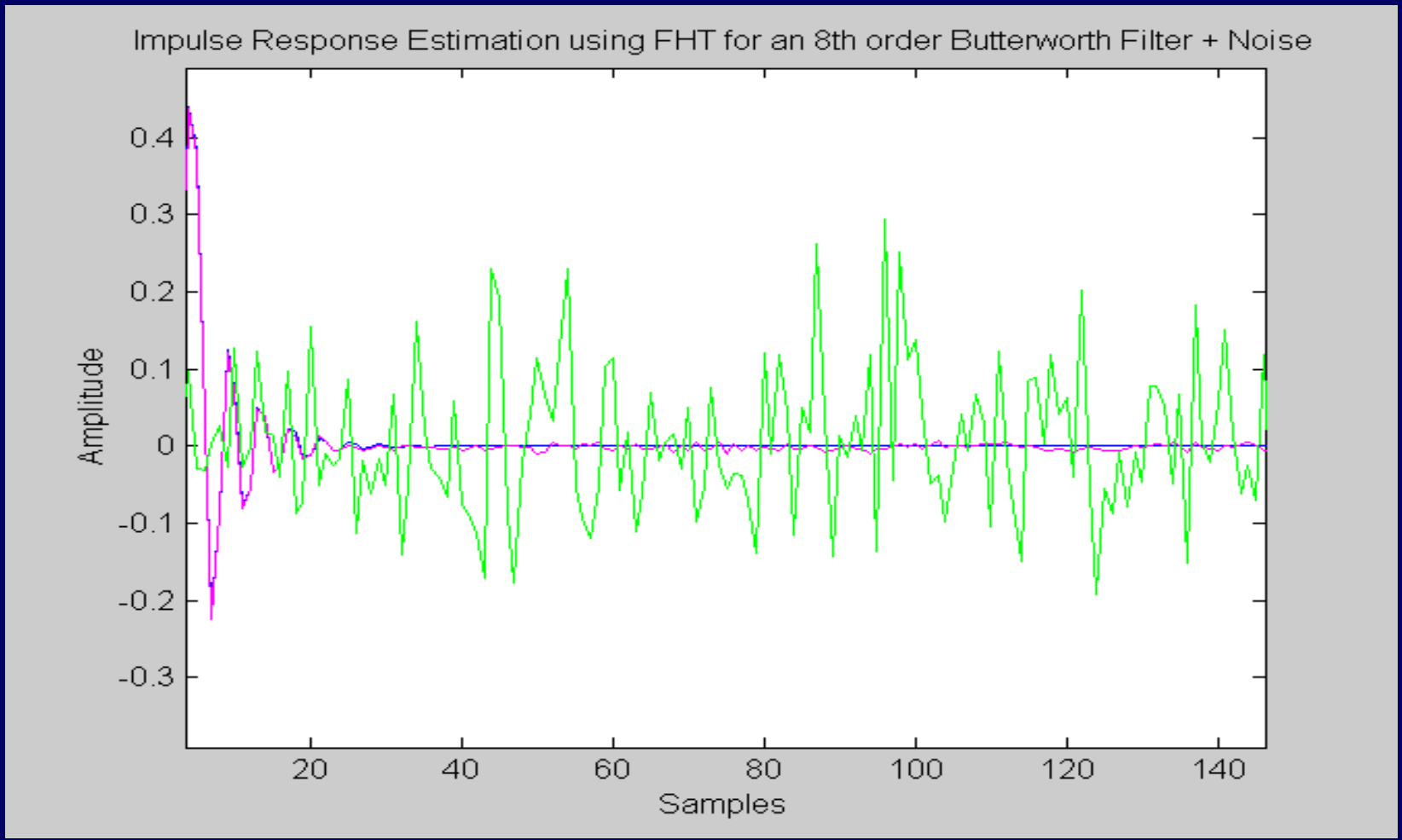
Application of FHT in Audio

- Case Study:
- Given an unknown linear system (speaker system), use the MLS technique to determine the impulse response of the system.
- The length of the sequence is 1024 points.
- The noise variance is 0.01.

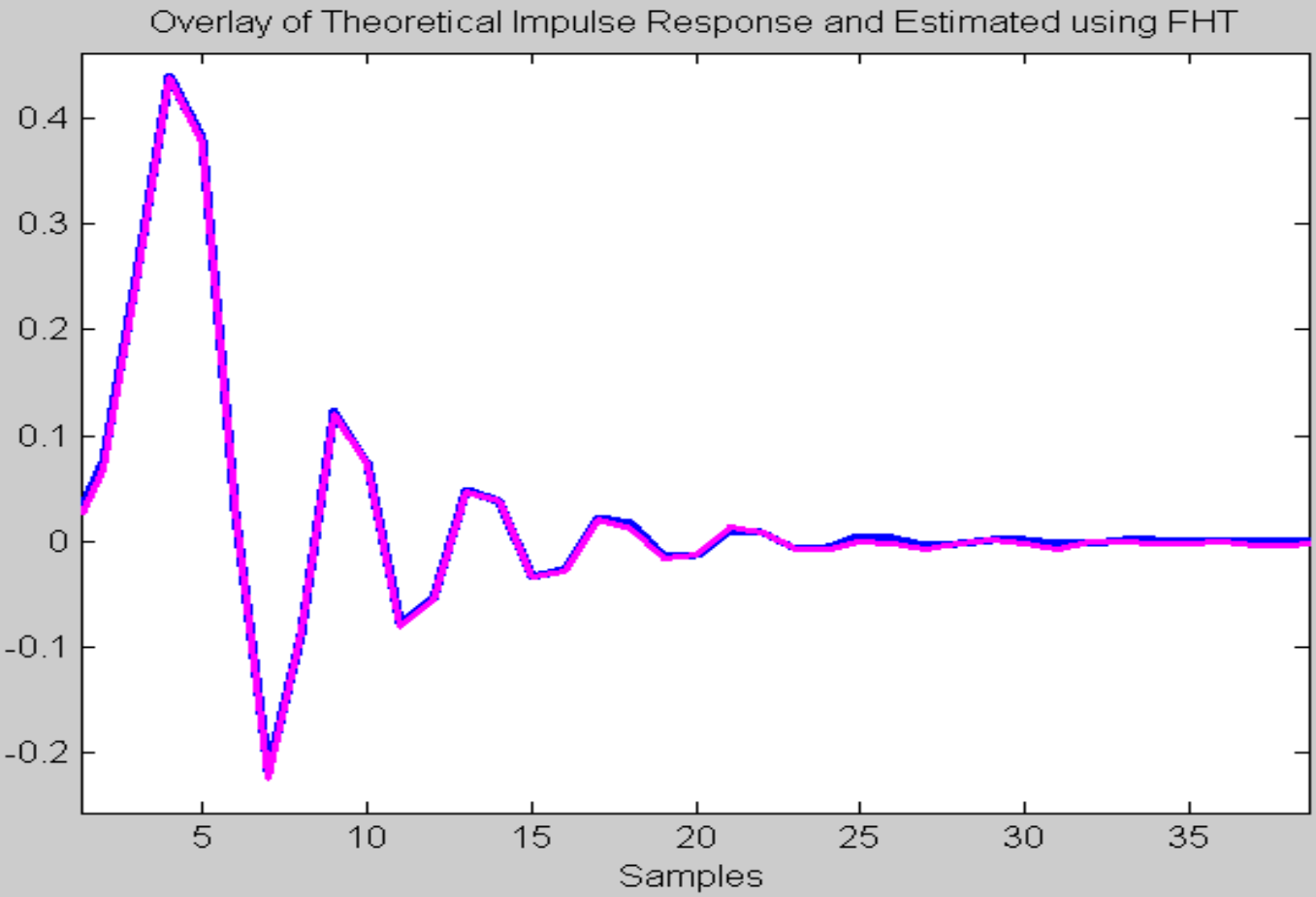
Results


Maximum Length Sequence
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
20
40
60
80
100
Samples

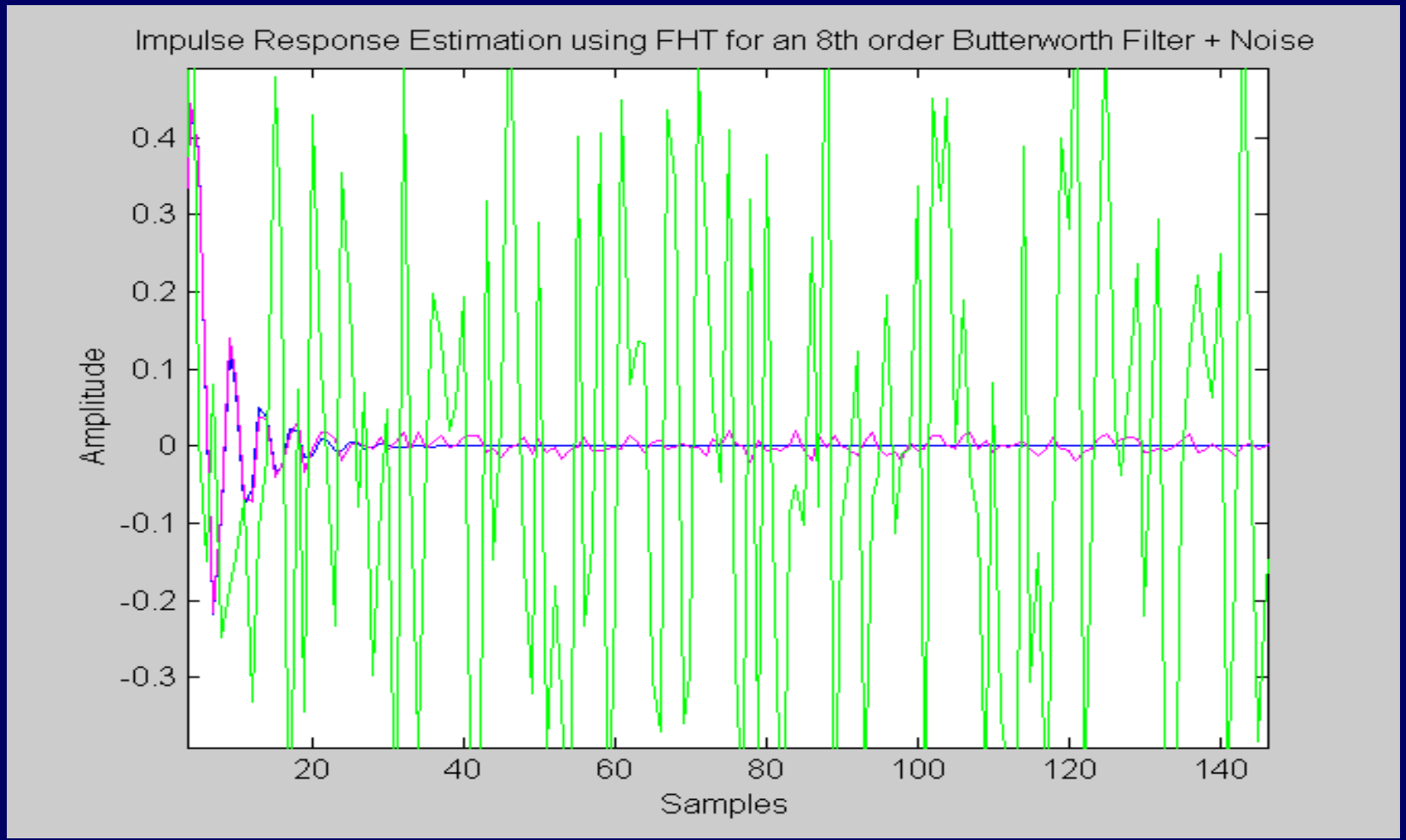
Results


Impulse Response Estimation using FHT for an 8th order Butterworth Filter + Noise
Amplitude
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
20
40
60
80
100
120
140
Samples

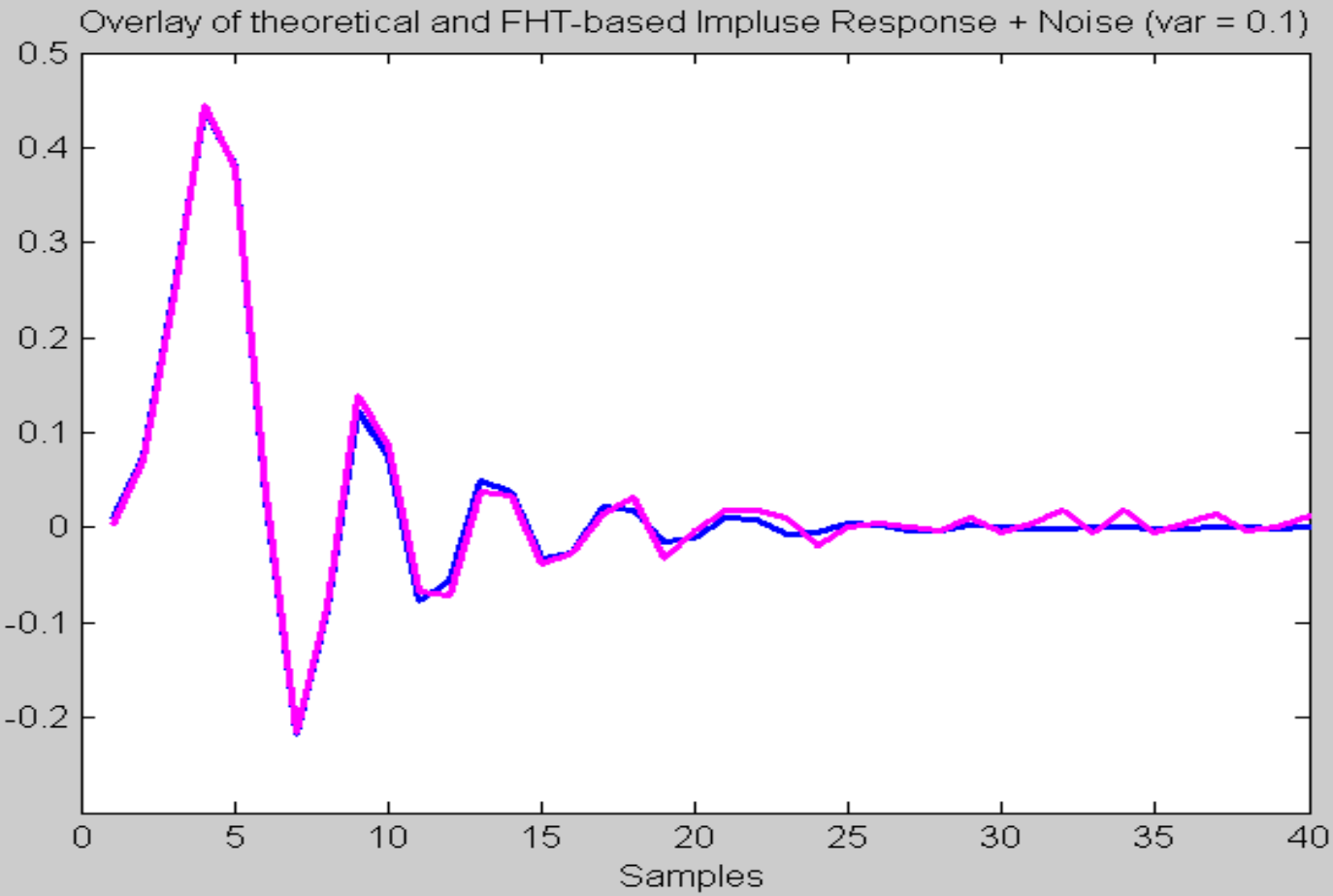
Results


Overlay of Theoretical Impulse Response and Estimated using FHT
0.4
0.3
0.2
0.1
0
-0.1
-0.2
5
10
15
20
25
30
35
Samples

Results


Impulse Response Estimation using FHT for an 8th order Butterworth Filter + Noise
Amplitude
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
20
40
60
80
100
120
140
Samples

Results


Overlay of theoretical and FHT-based Impluse Response + Noise (var = 0.1)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
25
30
35
40
Samples

Conclusions

- MLS is a good candidate for an input in input/output modeling.
- With the choice of MLS as input, the cross-correlation calculation is simplified.
- Use of Fast Hadamard Transform, speeds up the impulse response calculation.
- The technique is relatively immune to output noise.